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Non-classical approach to mathematical modeling of plate bending

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Abstract: This work is devoted to constructing a modified non-classical theory of plates without preliminary hypotheses about the distribution of the displacement vector and the deformation tensor on the basis of three-dimensional equations of the nonlinear elasticity theory of V.V. Novozhilov.

The proposed approach is applicable to deformation of non-canonical plates and shallow shells. The recurrent relations have been obtained; they will be further used in determining the ratios of interlayer variables in layered composites.

Key words: stress tensor, Cauchy tensor, stress-strain state, deformable body, recurrence relation.

Introduction

Plates are elements of thin-walled and thick-walled structures that are used in various fields of modern technology and construction of new facilities. Therefore, the determination of the stress-strain state of these elements is of great practical importance. At the same time, the solutions of some problems of statics, vibration and stability of these structures have been known even before a mathematical theory of elasticity was formulated [1].

A plate is a three-dimensional body where one of the dimensions is much smaller than the two others, so this body can be considered as two-dimensional with an important bearing capacity. Finding a solution of boundary-value problems for plates on the basis of the elasticity theory has significant difficulties, in this regard, a two-dimensional model is built to calculate this type of structure, which, in turn, takes into account the stress-strain state and a geometric feature.

In the elasticity theory there are various ways to reduce a three-dimensional problem to a two-dimensional one. These ways contribute to a significant simplification of the mathematical problem for which the number of independent variables is reduced to two ones. The special features of the distribution of stresses and strains in these bodies (plates, shells) are also taken into account. After reducing this type of boundary-value problems from three-dimensional to two-dimensional, their mathematical models are built on the basis of force and kinematic hypotheses [2].

Improved theories of shells and plates have significantly expanded the elasticity theory application in the engineering field [3,4,5]. In this paper, we will consider the reduction of a three-dimensional problem to a two-dimensional one using various versions of the improved theories of plates and shells. Also, the main difference between this work and the well-known works of the non-classical elasticity theory is that the coordinate origin is placed not in the plate middle where the Kirchhoff hypotheses are applied but starts at zero, which significantly changes the form of problem solving to a new version.

1. Problem statement

In the Cartesian coordinate system $Ox_1x_2x_3$, the stressed state of a three-dimensional body being under the action of surface and volume forces is described by the following nonlinear equation of motion [6,7,8]

$$\sigma_{ij'j} + X_i - \rho\ddot{U}_i = 0 \qquad (1)$$

where σ_{ij} is the stress tensor, U_i is the displacement vector, X_i are volume forces.

The defining relations between the stress tensor σ_{ij} and the finite strain tensor ε_{ij} have the form

$$\sigma_{ij} = C_{ijkl}\varepsilon_{kl}, \qquad (2)$$

and the geometric Cauchy relation between the displacement vector U_i and the components ε_{ij}

$$\varepsilon_{ij} = \frac{1}{2}\left(U_{i,j} + U_{j,i}\right)(3)$$

when the initial conditions

$$U_i|_{t=0} = U_i^0,\ \dot{U}_i\big|_{t=0} = V_i^0{=}o, \qquad (4)$$

and the boundary ones are met

$$U_i|_{\Sigma_1} = U_i^{\Sigma},\quad \sigma_{ij}n_j\big|_{\Sigma_2} = S_i, \qquad (5)$$

where U_i^{Σ} are the displacements specified at the boundary parts $\Sigma = \Sigma_1 + \Sigma_2$, U_i^0, V_i^0 are the characterizing initial conditions, n_j is the outer normal, ρ is the density, C_{ijkl} is the tensor of physical constants of materials [9,10].

In the above expressions there are operations of tensor analysis where the indices take the values 1,2,3 [11,12].

For isotropic materials, the following relationships are valid:

$$C_{ijkl} = \lambda\delta_{ij}\delta_{kl} + \mu\left(\delta_{ik}\delta_{jl} + \delta_{il}\delta_{jk}\right), \qquad (6)$$

where

$$\lambda = \frac{E\nu}{(1-2\nu)(1+\nu)},\ \mu = \frac{E}{2(1+\nu)}$$

λ, μ are the Lame coefficients, E is the elasticity modulus, ν is the Poisson ratio.

Thus, for the study of the stress-strain state of thick slabs and flexible plates, we have a general problem of the nonlinear elasticity theory. Due to the difficulties arising in solving nonlinear three-dimensional problems (1) - (6), in solid mechanics for the applied purposes the three-dimensional problems are usually reduced to two-dimensional ones.

2. Results and discussion

Let the Cartesian coordinate system Ox_1x_2z [17] be located in the initial plane of the slab with the constant thickness h and the surface loads $S_I^{\pm}$ at $z = x_3 = 0; h$.

We expand the displacement into a power series in z:

$$U_I = A_I + zB_I + z^2C_I + z^3D_I + \cdots, \quad U_z = a + zb + z^2\theta + \cdots, \qquad (7)$$

where $A_I, B_I, C_I, D_i, \ldots, a, b, \theta, \ldots$, are the unknown functions of coordinates x_1, x_2 and time t. Hereinafter, the indices have the value 1, 2. In order to simplify further calculations, we introduce the following integral values of the sought displacements

$u_I = \frac{1}{h}\int_0^h U_I dz\,, w = \frac{1}{h}\int_0^h U_z dz\,, \psi_I = \frac{3}{h^3}\int_0^h U_I z dz\,, V = \frac{3}{h^3}\int_0^h U_z z dz.$(8)

where u_I are the axial displacements, ψ_I are the shear angles of transverse fibers, transverse deflection w and parameter of thickness reduction V.

Substituting (7) into (8) we will have for the unknown coefficients in (7) the following expressions

$$\begin{cases} A_I = 4u_I - 2h\psi_I + \frac{h^2}{6}C_I + \frac{h^3}{5}D_I \\ B_I = 4\psi_I - \frac{6}{h}u_I - hC_I - \frac{9h^2}{10}D_I \end{cases} \begin{cases} a = 4w - 2hV + \frac{h^2}{6}\theta \\ b = 4V - \frac{6}{h}w - h\theta \end{cases}.$$

So, the displacements components can be represented as follows

$$\begin{cases} U_I = 4\left(1 - \frac{3}{2}\left(\frac{z}{h}\right)\right)u_I + 2(2z-h)\psi_I - \frac{1}{2}\Phi_1(z)C_I - \frac{3h^2}{5}\Phi_2(z)D_I \\ U_z = 4\left(1 - \frac{3}{2}\left(\frac{z}{h}\right)\right)w + 2(2z-h)V - \frac{1}{2}\Phi_1(z)\theta \end{cases} \quad (9)$$

where $\Phi_1(z) = -\left(\frac{h^2}{3} - 2hz + 2z^2\right)$, $\Phi_2(z) = -\left(\frac{h}{3} - \frac{3z}{2} + \frac{5z^3}{3h^2}\right)$, at the same time the following relations are met:

$$\int_0^h \Phi_I(z)dz = 0, \quad \int_0^h \Phi_I(z)zdz = 0, \quad (10)$$

which can be easily checked.

The plate bending is accompanied by the predominance of one of the components of the displacement vector, namely, the condition $U_Z >> U_I$ is satisfied, i.e. lateral displacement as compared to the rest.

In this regard, in expressions (1) - (6), the nonlinear terms are usually neglected for the longitudinal components of the displacement vector U_i when plate bending. The nonlinear components of the U_Z derivative with respect to the normal z coordinate are also assumed to be negligibly small.

Then the component of the Lagrange strain tensor will have

$$\varepsilon_{IJ} = \frac{1}{2}(U_{I,J} + U_{J,I}), \quad \varepsilon_{I3} = \frac{1}{2}(U_{I,3} + U_{3,I}), \quad \varepsilon_{33} = U_{3,3}. \quad (11)$$

The Boundary conditions (5) can be written in the following form with the substitution in indices 3 => z:

$$\sigma_{ZI} = S_I^{\pm} \quad \text{for } z = 0; h \quad (12)$$

$$\sigma_{ZZ} = S_Z^{\pm} - U_{Z,I}S_I^{\pm} \quad \text{for } z = 0; h \quad (13)$$

The boundary conditions (5) were taken into account in (13). Taking into account (9), for elastic isotropic plates, the components of the longitudinal stress tensor will have the following form:

$$\sigma_{IJ} = \lambda\Big\{4\left(1 - \frac{3}{2}\left(\frac{z}{h}\right)\right)u_{K,K} + 2(2z-h)\psi_{K,K} - \frac{1}{2}\Phi_1(z)C_{K,K} - \frac{3h^2}{5}\Phi_2(z)D_{K,K} + \Big(-\frac{6}{h}w + 4V + (2z-h)\theta\Big) + \frac{1}{2}\Big(4\left(1 - \frac{3}{2}\left(\frac{z}{h}\right)\right)w + 2(2z-h)V - \frac{1}{2}\Phi_1(z)\theta\Big)_{,K}\Big(4\left(1 - \frac{3}{2}\left(\frac{z}{h}\right)\right)w + 2(2z-h)V - \frac{1}{2}\Phi_1(z)\theta\Big)_{,K}\Big\}\delta_{IJ} + \mu\Big\{4\left(1 - \frac{3}{2}\left(\frac{z}{h}\right)\right)(u_{I,J} + u_{J,I}) + 2(2z-h)(\psi_{I,J} + \psi_{J,I}) - \frac{1}{2}\Phi_1(z)(C_{I,J} + C_{J,I}) - \frac{3h^2}{5}\Phi_2(z)(D_{I,J} + D_{J,I}) + \Big(4\left(1 - \frac{3}{2}\left(\frac{z}{h}\right)\right)w + 2(2z-h)V - \frac{1}{2}\Phi_1(z)\theta\Big)_{,I}\Big(4\left(1 - \frac{3}{2}\left(\frac{z}{h}\right)\right)w + 2(2z-h)V - \frac{1}{2}\Phi_1(z)\theta\Big)_{,J}\Big\} \quad (14)$$

The following boundary condition (15)

$$\sigma_{ZI} = 2\mu\varepsilon_{IZ} = S_I^{\pm} \quad (15)$$

is satisfied because

$$\left[-\frac{6}{h}u_I + 4\psi_I + \frac{h^2}{6}\theta_{,I} + \frac{1}{2}\left(\left(\frac{21}{10} \mp \frac{9}{10}\right)h^2 D_I - (2 \mp 4)w_{,I}\right)\right] \pm 2h[C_I + 2V_{,I}] = \frac{S_I^{\pm}}{\mu}$$

We introduce new notation

$$Y_I^- = \frac{S_I^+ - S_I^-}{h} \text{ и } Y_I^+ = \frac{S_I^+ + S_I^-}{2}$$

then for $z = 0; h$, we obtain the following

$$\begin{cases} C_I = \frac{Y_I^-}{2\mu} - \frac{5Y_I^+}{2h\mu} - \frac{15}{h^2}u_I + \frac{10}{h}\psi_I + \frac{11}{2h}w_{,I} - 2V_{,I} + \frac{5h}{12}\theta_{,I} \\ D_I = \frac{5}{3h^2}\left(\frac{Y_I^+}{\mu} + \frac{6}{h}u_I - 4\psi_I - w_{,I} - \frac{h^2}{6}\theta_{,I}\right) \end{cases} \quad (15)$$

Taking into account (15), the component of the displacement vector can be written in the following form

$$\begin{cases} U_I = \left(\frac{7}{2} - 15\left(\frac{z}{h}\right)^2 + 10\left(\frac{z}{h}\right)^3\right)u_I + \left(-\frac{5h}{3} + \frac{10z^2}{h} - \frac{20z^3}{3h^2}\right)\psi_I - \\ -\frac{1}{2}\Phi_1(z)\left(\frac{Y_I^-}{2\mu} - \frac{5Y_I^+}{2h\mu} + \frac{11}{2h}w_{,I} - 2V_{,I} + \frac{5h}{12}\theta_{,I}\right) - \Phi_2(z)\left(\frac{Y_I^+}{\mu} - w_{,I} - \frac{h^2}{6}\theta_{,I}\right) \\ U_z = 4\left(1 - \frac{3}{2}\left(\frac{z}{h}\right)\right)w + 2(2z - h)V - \frac{1}{2}\Phi_1(z)\theta \end{cases} \quad (16)$$

Based on (16) for the normal stress σ_{ZZ}, the following expression can be obtained

$$\sigma_{ZZ} = \lambda\left\{\left(\frac{7}{2} - 15\left(\frac{z}{h}\right)^2 + 10\left(\frac{z}{h}\right)^3\right)u_{K,K} + \left(-\frac{5h}{3} + \frac{10z^2}{h} - \frac{20z^3}{3h^2}\right)\psi_{K,K} - \frac{1}{2}\Phi_1(z)\Delta\left(\frac{11}{2h}w - 2V + \frac{5h}{12}\theta\right) + \Phi_2(z)\Delta\left(w + \frac{h^2}{6}\theta\right) + \frac{1}{2}\left(\left(4 - 6\left(\frac{z}{h}\right)\right)w + 2(2z-h)V - \frac{1}{2}\Phi_1(z)\theta\right)_{,K}\left(\left(4 - 6\left(\frac{z}{h}\right)\right)w + 2(2z-h)V - \frac{1}{2}\Phi_1(z)\theta\right)_{,K} + \frac{1-\nu}{\nu}\left(-\frac{6}{h}w + 4V + (2z-h)\theta\right) - \frac{1}{2}\Phi_1(z)\left(\frac{Y_{I,I}^-}{2\mu} - \frac{5Y_{I,I}^+}{2h\mu}\right) - \Phi_2(z)\frac{Y_{I,I}^+}{\mu}\right\}$$

(17)

The boundary condition (13) with taking into account (17) is satisfied when the following nonlinear differential equation partial derivatives is fulfilled

$$\begin{cases} D\left(\Delta V - \frac{1}{4\mu}Y_{I,I}^-\right) - \frac{1}{2}G\left(u_{K,K} + \frac{h}{2}\Delta w + \left(5w_{,K}w_{,K} + 2h^2V_{,K}V_{,K} + \frac{h^4}{72}\theta_{,K}\theta_{,K} - 6hw_{,K}V_{,K} + \frac{h^2}{6}w_{,K}\theta_{,K}\right) - \frac{1-\nu}{\nu}\left(\frac{6}{h}w - 4V\right)\right) + \frac{(1-2\nu)}{4\nu}Z_Z^+h = 0 \\ D\left(\Delta\theta - \frac{6}{h^2\mu}Y_{I,I}^+\right) + G\left(\frac{30}{h}u_{K,K} - 20\psi_{K,K} + \Delta w + \left(\frac{36}{h}w_{,K}w_{,K} - 24w_{,K}V_{,K} + 6hw_{,K}\theta_{,K} - 4h^2V_{,K}\theta_{,K}\right)\right) - 12G\frac{1-\nu}{\nu}\theta + \frac{3(1-2\nu)}{\nu}Z_Z^- = 0 \end{cases}$$

(18)

where $= \frac{Eh^3}{12(1+\nu)}$, $G = \frac{Eh}{2(1+\nu)}$, Δ is the Laplace operator.

$$Z_Z^+ = \frac{S_Z^+ - \left(-2w_{,i} + 2hV_{,i} + \frac{h^2}{6}\theta_{,i}\right)S_i^+ + S_Z^- - \left(4w_{,i} + -2hV_{,i} + \frac{h^2}{6}\theta_{,i}\right)S_i^-}{2}$$

$$Z_Z^- = S_Z^+ - \left(-2w_{,I} + 2hV_{,I} + \frac{h^2}{6}\theta_{,I}\right)S_I^+ - S_Z^- + \left(4w_{,I} + -2hV_{,I} + \frac{h^2}{6}\theta_{,I}\right)S_I^-$$

After satisfying the boundary condition (15) with taking into account (14) and (19), we obtain for tangential stresses

$\sigma_{zi} = \mu\left\{-\frac{6}{h}u_i + 4\psi_i + w_{'i}\right\}f(z) + \frac{z\mu}{6}(z-h)\theta_{'i} + Y_i^-\left(z-\frac{h}{2}\right) + Y_i^+(1-f(z))$ где

$f(z) = \frac{5z}{h}\left(1-\frac{z}{h}\right)$ (19)

The analytical expressions obtained in the geometric nonlinear form (14)-(19) for the components of the displacement vector and the Cauchy stress tensor satisfy the boundary conditions (12)-(13).

The components of the displacement vector (19) and the symmetric stress tensor depend on the unknown integral quantities $u_I, \psi_I, w, V, \theta$ that are the functions of the coordinates x_1, x_2 and t. For a closed system with respect to these unknown resolving equations to be obtained, the following integral quantities of the stress tensor components are introduced into consideration: normal forces $N_{ij} = \int_0^h \sigma_{ij}dz$, shear forces $Q_i = \int_0^h \sigma_{ij}dz$, inner bending moments $M_{ij} = \int_0^h \sigma_{ij}zdz$.

Substituting the stress tensor components (14)-(19) into the obtained integral expressions, we will have

$$N_{IJ} = G\left[\frac{2\nu}{(1-2\nu)}\left\{u_{K,K} - \frac{6}{h}w + 4V + 2w_{'K}w_{'K} + \frac{2h^2}{3}V_{'K}V_{'K} + \frac{h^4}{360}\theta_{'K}\theta_{'K} - 2hw_{'K}V_{'K}\right\}\delta_{IJ} + u_{I,J} + u_{J,I} + 4w_{'I}w_{'J} + \frac{4h^2}{3}V_{'I}V_{'J} + \frac{h^4}{180}\theta_{'I}\theta_{'J} - 2hw_{'I}V_{'J} - 2hw_{'J}V_{'I}\right] \quad (20)$$

$$M_{IJ} = D\left[\frac{\nu}{(1-2\nu)}\left\{4\psi_{K,K} - \frac{36}{h^2}w + \frac{24}{h}V + 4\theta + \frac{6}{h}w_{'K}w_{'K} + 20hV_{'K}V_{'K} + \frac{h^3}{60}\theta_{'K}\theta_{'K} - 8w_{'K}V_{'K} - \frac{2h}{5}w_{'K}\theta_{'K} + \frac{4h^2}{15}V_{'K}\theta_{'K}\right\}\delta_{IJ} + 2(\psi_{I,J} + \psi_{J,I}) + \frac{6}{h}w_{'I}w_{'J} + hV_{'I}V_{'J} + \frac{h^3}{60}\theta_{'I}\theta_{'J} + 20w_{'I}V_{'J} + 20w_{'J}V_{'I} - \frac{h}{5}w_{'I}\theta_{'J} - \frac{h}{5}w_{'J}\theta_{'I} + \frac{2h^2}{15}V_{'I}\theta_{'J} + \frac{2h^2}{15}V_{'J}\theta_{'I}\right] \quad (21)$$

$$Q_I = Gk^2\left\{-\frac{6}{h}u_I + 4\psi_I + w_{'I} - \frac{h^2}{30}\theta_{'I}\right\} + \frac{h}{6}Y_I^+, k^2 = \frac{5}{6} \quad (22)$$

The linearized record of normal forces, inner bending moments and shearing forces (20), (21), (22) will have the form

$$N_{IJ} = G\left[\frac{2\nu}{(1-2\nu)}\left\{u_{K,K} - \frac{6}{h}w + 4V\right\}\delta_{IJ} + u_{I,J} + u_{J,I}\right] \quad (23)$$

$$M_{IJ} = D\left[\frac{\nu}{(1-2\nu)}\left\{4\psi_{K,K} - \frac{36}{h^2}w + \frac{24}{h}V + 4\theta\right\}\delta_{IJ} + 2(\psi_{I,J} + \psi_{J,I})\right] \quad (24)$$

$$Q_I = Gk^2\left\{-\frac{6}{h}u_I + 4\psi_I + w_{'I} - \frac{h^2}{30}\theta_{'I}\right\} + \frac{h}{6}Y_I^+, \ k^2 = \frac{5}{6} \quad (25)$$

It should be noted that the so-called shear coefficient k^2 is determined as a consequence of the boundary condition fulfillment for tangential stresses. In the improved elasticity theories of the Timoshenko type [13,14,15,16,17], the special experiments are carried out to determine this coefficient that is almost 5/6.

To obtain the equations of motion with taking into account the deformation of thick slabs or flexible plates and usually based on the d'Alembert principle for the elementary object under consideration, the equations of motion will be constructed by the method of projection of all outer and inner forces onto the coordinate ones, in each case separately. But in this case, the connection with the three-dimensional equations of motion in the theory of elasticity is lost. And these equations are the most accurate for slabs. Therefore, for the equations of motion to be obtained for elastic plates, we perform the integration procedure (1) taking into account (14) with respect to z within the range [0; h]:

$$\begin{cases} N_{IJ,J} + p_I - m\ddot{u}_I = 0 \\ M_{IJ,J} - Q_I + m_I - \frac{h^2}{3} m\ddot{\psi}_I = 0 \\ Q_{I,I} + \left(\left(4N_{KI} - \frac{6}{h} M_{KI}\right) w_{,I}\right)_{,I} + 2\left((2M_{KI} - hN_{KI})V_{,I}\right)_{,I} + \left(\left(\frac{h^2}{6} N_{KI} - hM_{KI}\right)\theta_{,I}\right)_{,I} + S_Z^+ - S_Z^- - m\ddot{w} = 0 \end{cases} \quad (26)$$

where $p_I = S_I^+ - S_I^- + \int_0^h X_I dz$, $m_I = hS_I^+ + \int_0^h X_I dz$, are the distributed outer forces and moments; $m = \rho h$ is the running weight. Here, the volume force X_3 is neglected, and the density ρ along the thickness is considered to be constant.

It should be stressed here that in the resulting system of equations of motion (26), the first two equations do not formally differ in appearance from the improved equations of S.P. Tymoshenko [13]. In this case, the equations of motion (26) with taking into account (18) become closed with respect to the following integral quantities: $u_I, \psi_I, w, V, \theta$.

At the plate edges there are usually three types of fastening that in terms of stresses and displacements can be written as [18]

На торцах пластин обычно имеют место три типа закрепления, которые в терминах напряжений и перемещений можно записать в виде [18]

I. $\sigma_{IJ} n_J\big|_\Sigma = 0,\ U_Z|_\Sigma = 0$ - knuckle joint

II. $U_I|_\Sigma = 0,\ U_Z|_\Sigma = 0$ - hard pinching (27)

III. $\sigma_{IJ} n_J\big|_\Sigma = 0,\ \sigma_{ZI}|_\Sigma = 0$ - free edge

The above boundary conditions, in terms of integral quantities, are written in the following form, respectively, [19,20]

I. $N_{IJ} n_J\big|_\Sigma = 0, M_{IJ} n_J\big|_\Sigma = 0, w|_\Sigma = 0, V|_\Sigma = 0, \theta|_\Sigma = 0$

II. $u_I|_\Sigma = 0, \psi_I|_\Sigma = 0, w|_\Sigma = 0, V|_\Sigma = 0, \theta|_\Sigma = 0$ (28)

III. $N_{IJ} n_J\big|_\Sigma = 0, M_{IJ} n_J\big|_\Sigma = 0, Q_I n_I|_\Sigma = 0, V_{,I} n_I|_\Sigma = 0, \theta_{,I} n_I|_\Sigma = 0$

In the boundary conditions obtained for the first two types, there is no normal displacement at the edges of the slab; therefore, for V, θ the basic boundary conditions are satisfied; for the third type it is assumed in (28) that there is no gradient for compression along the free slab edges. It should be noted that the typical boundary conditions are formulated here. Other types of boundary conditions can be obtained either by a linear combination of the above conditions or formulated in each case separately. The initial conditions (2) with taking into account (16) can be written in a similar way in terms of integral quantities.

Neglecting the nonlinear terms in expressions (1)-(19), we will have a linear theory of slabs for which a system of the resolving equations will be as follows

$$\begin{cases} G\left[\Delta u_I + \frac{1}{(1-2\nu)} u_{K,KI} - \frac{2\nu}{(1-2\nu)}\left(\frac{6}{h} w_{,J} - 4V_{,J}\right)\right] + p_I - m\ddot{u}_I = 0 \\ 2D\left[\Delta\psi_I + \frac{1}{(1-2\nu)}\psi_{K,KI} - \frac{2\nu}{(1-2\nu)}\left(\frac{9}{h^2} w_{,J} - \frac{6}{h} V_{,J} - \theta_{,J}\right)\right] + Gk^2\left\{\frac{6}{h} u_I - 4\psi_I - w_{,I} + \frac{h^2}{30}\theta_{,I}\right\} + \frac{h}{6} Y_I^+ + m_I - \frac{h^2}{3} m\ddot{\psi}_I = 0 \\ Gk^2\left\{-\frac{6}{h} u_{I,I} + 4\psi_{I,I} + \Delta\left(w - \frac{h^2}{6}\theta\right)\right\} + \left(\frac{h}{6} Y_I^+\right)_{,I} + S_Z^+ - S_Z^- - m\ddot{w} = 0 \\ \frac{1}{2} G\left(u_{K,K} + \frac{h}{2}\Delta w - \frac{1-\nu}{\nu}\left(\frac{6}{h} w - 4V\right)\right) - D\left(\Delta V - \frac{1}{4\mu} Y_{I,I}^-\right) = \frac{(1-2\nu)}{4\nu} Z_Z^+ h \\ G\left(\frac{30}{h} u_{K,K} - 20\psi_{K,K} + \Delta w\right) - D\left(\Delta\theta - \frac{6}{h^2\mu} Y_{I,I}^+\right) + 12G\frac{1-\nu}{\nu}\theta = \frac{3(1-2\nu)}{\nu} Z_Z^- \end{cases}$$

(29)

Hereinafter, we assume that only the transverse surface load acts $S_Z^+ = q(x_1, x_2, t)$, $S_Z^- = S_I^\pm = 0$, и $p_I = m_I = 0$. Then, introducing the potentials $u_I = u_{,I}$, $\psi_I = \psi_{,I}$ instead of (29), we will have the following system of five differential equations in partial derivatives

$$\begin{cases} G\left[\Delta u - \frac{\nu}{1-\nu}\left(\frac{6}{h}w - 4V\right)\right] - \frac{1-2\nu}{2(1-\nu)}m\ddot{u} = 0 \\ 2D\left[\Delta\psi - \frac{\nu}{1-\nu}\left(\frac{9}{h^2}w - \frac{6}{h}V - \theta\right)\right] + \frac{1-2\nu}{2(1-\nu)}Gk^2\left\{\frac{6}{h}u - 4\psi - w + \frac{h^2}{6}\theta\right\} - \frac{1-2\nu}{1-\nu}\frac{h^2}{6}m\ddot{\psi} = 0 \\ k^2G\Delta\left\{4\psi - \frac{6}{h}u + w - \frac{h^2}{6}\theta\right\} + q - m\ddot{w} = 0 \\ D\Delta V - \frac{1}{2}G\left(\Delta\left(u + \frac{h}{2}w\right) - \frac{1-\nu}{\nu}\left(\frac{6}{h}w - 4V\right)\right) + \frac{(1-2\nu)}{8\nu}qh = 0 \\ D\Delta\theta - G\Delta\left(\frac{30}{h}u - 20\psi + w\right) - 12G\frac{1-\nu}{\nu}\theta + \frac{3(1-2\nu)}{\nu}q = 0 \end{cases} \quad (30)$$

The system of equations (30) can be further applied as a recurrence relation for solving the problem of the equilibrium of an inhomogeneous plate with an arbitrary number of layers.

Conclusions

1. This work is devoted to constructing a modified non-classical theory of plates without preliminary hypotheses about the distribution of the displacement vector and strain tensor on the basis of three-dimensional equations of the nonlinear theory of elasticity.
2. The main difference of this work from the well-known works of the non-classical theory of elasticity is that a three-dimensional deformable body with a constant thickness h and surface loads $S_I^\pm$ for $z = x_3 = 0; h$ has been considered.
3. The proposed approach is applicable to deformation of layered plates and shallow shells of non-canonical shape. A recurrent relationship has been obtained; it will be further used in determining the ratios of interlayer variables in layered composites.

Reference

1. Lyav A. Mathematical theory of elasticity. M.; L.: ONTI, 1935, 674 p.
2. Annin B.D., Volchkov Yu.M. Non-classical models of the theory of plates and shells. // PMTF. 2016 Vol.57, No. 5 pp. 5-14.
3. Ambartsumyan S.A. Theory of anisotropic plates. Moscow: Nauka, 1987, 360 p.
4. Galinsh A.K. Calculation of plates and shells according to refined theories. Studies of the theory of plates and shells. Issue 6, Kazan: Publishing house of Kazan University, 1970, p.23-64.
5. Grigoglyuk E.I., Selezov I.T. Non-classical theories of vibrations of rods, plates and shells. // Results of science and technology M.: 1973, volume 5, 230 p.
6. Novozhilov V.V. Theory of elasticity. L.: Sudpromgiz, 1958, 370 s
7. Novozhilov V.V. Fundamentals of the nonlinear theory of elasticity. M.; L.: Gostekhizdat, 1948, 201 p.
8. Kirchhoff G. Vorlesungen uber mathematishe Phisik. Bd 1. Mechanik. Leipzig: B. G. Teubner, 1876.

9. Akhmedov A.B. Non-classical problems of plates and shallow shells and the development of numerical solution methods. // Dissertation Doctor. phys.-math. Sciences, Tashkent, 2016, 243s.
10. Akhmedov A.B. Non-classical theory of bending of laminated composites. // Bulletin of TSTU, Tashkent, 2015, № 2, p.10-17.
11. Pobedrya B.E. Lectures on tensor analysis. M.: Publishing house of Moscow State University, 1974, 206s.
12. Novozhilov V.V. Nonlinear problem for a plane continuous medium in Euler coordinates. Vestn. Leningrad. Univ.. 1972, No.19, p. 103-105 (together with Morozov N.F.).
13. Timoshenko S.P. Plates and shells / Timoshenko S.P., Votsiovskiy-Krieger S. M.: Fizmatgiz, 1963, 636p.
14. Novozhilov V.V. Statics of thin-walled elastic shells. L.: Gostekhteoretizdat, 1947.
15. Novozhilov V.V. On the relationship between stresses and deformations in a nonlinear elastic medium.Applied mathematics and mechanics. 1951, vol. 15, issue 2, pp. 183-194.
16. Voigt W. Theoretische Studien uber die Elastizitats verhaltnisse der Kristalle. - Abh. Ges. Wiss. Gottingen, 1887, Bd. 54.
17. Vlasov V.Z. General theory of shells and its applications in technology. M.; L.: Gostekhteoretizdat, 1949.
18. Novatsky V. Theory of elasticity. M.: Mir, 1975.
19. Akhmedov A.B. Numerical solution of spectral problems. Tashkent: FAN, 2012, 118 p.
20. Akhmedov A.B. A qualitative analysis of the theories of plates made of composite materials. Uzbek journal of the problem of mechanics, Tashkent, 2011, No. 1, P.81-84.