

# Method of determination of car movement under the fair wind on the sections of yard hump braking positions

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**Abstract.** Hitherto there have not been investigated zones of yard hump braking positions under the action of fair wind of small magnitude. There have not been analytically defined a car braking path on the sections of hump braking positions.

**Purpose.** The purpose of the investigation is to construct a mathematical model of the car movement on the sections of the hump braking positions, to derive a braking path formula and calculate its values for a specific example.

**Methods:** Mathematical description of car movement on the sections of hump braking positions is performed with the help of the theorem on the change in kinetic energy for a non-free material point in a finite form. For validity assessment of the analytical formula of the car braking path there has been used an elementary physics path formula.

**Results.** For the first time there has been obtained a car braking path formula. Car braking path is directly proportional to a square of the initial velocity, the grade of hump track profile and inversely proportional to the coefficient of sliding friction. According to the force ratios in the deceleration zone, there has been defined linear acceleration under uniformly decelerated car motion. According to the value of the initial velocity and acceleration and on the basis of the elementary physics formula there has been found car motion velocity before the complete stop of the car in the braking zone. With the help of the car on velocity there has been derived a formula for defining car braking time before the complete stop of the car. Calculations have shown that car braking path based on the formula put forward by the authors of the article and elementary physics formulas under the same initial velocity values give similar results. This confirms the correctness of the obtained formula. **Practicability.** The

results of the investigation on defining car braking path in the braking zone can be used in designing hump track profile.

**Key words.** Railway, station, hump, car, braking position, braking path on braking positions.

**Introduction.** This article is a continuation of a series of publications on the problem of calculating and designing a hump track profile [1 – 34].

It is significant that in [7, 10] for determination of hump energy height in a gravity yard  $h_{\text{hump}}$  one and the same method is used. In this method, it is assumed that at any point of the grade plane the energy of the rolling body of mass  $M$  is equal to the sum of the potential  $E_p$  and kinetic energy  $E_c$ . It is supposed that this energy is executed on the work to overcome resistance forces to the motion  $A_r$ , i.e.  $E_c + E_p = A_r$  (q.v. page in [7] and formula (6) in [10]). Hence hump energy height yard  $h_{\text{hump}}$  is determined. However, this contradicts the law of energy conservation [12,14]. Up to this date the designing of yard hump energy height  $h_{\text{hump}}$  has been performed according to the so-called notion “the power of braking position  $h_{\text{br}}$ ” or «the power of braking tools”  $h_{\text{brt}}$ ” [1, 2, 7, 10, 17].

Braking position power  $h_{\text{br}}$  is chosen according to [17]. Yard hump energy height  $h_{\text{hump}}$  within the bounds of its effective length (from the hump crest (HC) to the design point (DP)) is determined as the sum of three profile heights of design sections (q.v. Figure 5.1 in [17]): the higher section (between the hump crest and the beginning of the first braking position (1BP))  $h_{\text{hump1}}$ , the medium section (before the beginning of the 1BP and the beginning of the second beam braking position (2BP))  $h_{\text{hump2}}$  and the lower section (before the beginning of the beam 2BP and DP)  $h_{\text{hump3}}$ . At that, values  $h_{\text{hump1}}$ ,  $h_{\text{hump2}}$  and  $h_{\text{hump3}}$  are calculated according to the empirical formula (q. v. formula (5.1) in [17]). This formula consists of three items. The first formula (5.1) in [17] is obtained in the absence of the constraint force between car wheels and rail tracks (i.e. for an ideal constraint) when all car kinetic energy during its relocation from DP at some distance is equal to the work of the gravity force component towards the car relocation direction ( $G\sin\alpha$ ) along rail tracks [17]. The second and the third formulas (5.1) in [17] are allegedly losses of specific energy executed on overcoming major specific resistance to motion and switch and curves resistance in the bounds of corresponding sections  $h_{\text{hump1}}$ ,  $h_{\text{hump2}}$  and  $h_{\text{hump3}}$  [17]. Otherwise, formulas for determination  $h_{\text{hump1}}$ ,  $h_{\text{hump2}}$  and  $h_{\text{hump3}}$  belong to the class of non-ideal constraints. The same calculation method is observed in [7, 10]. According to the first formula (5.1) in [27] car wheels slide along rail tracks. It does not agree with the actual motion of car wheels along rail tracks because the latter cannot be presented in the form of ideal constraints. According to the second and third formula (5.1) in [17], car wheels roll without sliding along rail tracks. In reality the profile of the yard hump (gradient plane) is a non-ideal (non-smooth or/and with friction) surface and the car with cargo moves the yard hump profile with initial velocity  $v_0$ , equal to zero. (i.e.  $v_0 \neq 0$ ).

Despite this fact, all the research results on hump designing in [1, 2, 7 – 11, 15 – 19] have been obtained:

*first*, with the regard for the inertia of rolling parts (wheel pairs) including sections of braking positions where there is only translatory motion of wheel pairs. Here it is inadmissible to take into account the inertia of rolling parts on the sections

of braking positions where only pure sliding of wheel pairs relative to rail tracks and retarder brake beams takes place;

*second*, with the regard for hump profile as a *non-ideal surface*;

*third*, the speeds on all hump sections including braking positions are determined according to formula  $v = \sqrt{2g'h}$  (where  $g'$  – is the acceleration of a freely falling body with the regard for rolling parts inertia) (q. v. page 186 in [1]), as applied to an *ideal surface* which is unacceptable.

As can be seen, fallaciousness of defining hump energy height  $h_{\text{hump}}$  is in using an incompatible in physical sense notion of ideal\* constraint as applied to the solution of a hump problem, where constraints between car wheel pairs and rail tracks belong exclusively to the class of *non-ideal* ones.

Until now, the authors of the article [17-18] considered the solution of the problem on defining the time and braking path of a braked car to be an intractable one. In [16-18] just as in the existing method of designing and technological calculations in [14-16] it is recommended to solve this problem through the notion of “braking positions power” which is an erroneous concept. (q. v. the subtrahend in formula (4) in [13]). Here the error consists in using the formula of free fall of the body  $v = \sqrt{2g'h}$ , applicable only for an ideal constraint [19].

Hence, it is clear that the problem of mathematical modeling of hump braking sections has not been solved yet. Therefore, this problem is urgent for transport science, particularly, for railway transport.

**Research Purpose.** To construct a mathematical model of car motion on the sections of hump yard braking positions. To derive a finite analytical formula of a car braking path

**Problem formulation.** It is required to perform an example of calculation of a car braking path on the sections of hump braking positions and to compare calculation results with elementary physics path formula.

**Research method.** The problem was solved by means of application of theorem on kinetic energy change for a non-free material point in a finite form [15,16,18]. Applicability of an elementary physics path formula is proved on the basis of D’Alembert principle [18,19].

**Man-made assumptions.** Let us assume that any car points, including its mass center  $C_w$  and wheel pair mass center  $C$  in braking zones on the sections of braking positions move with the same velocities equal to a car inlet velocity to this zone  $v_{e.b} = v_o = v_{ib}$ , i.e.  $v_{C_w} = v_C = v_{e.b} = v_o = v_{ib} = \text{const} > 0$ . Therefore, the motion of the car with cargo  $G$  in braking zones can be looked upon as the motion of a system of material points assuming that all its mass  $M_w$  is concentrated in its mass center  $C_w$ . In this case, for the computing origin of the fixed reference system (here it is not presented) we take the position in which point  $C_w$  was at the beginning of its braking. That is why, for the origin of time computing, i.e. for the initial moment  $t = 0$ , we will take the moment of the beginning of car braking with car moving along the hump grade with constant velocity  $v_{ib} = v_{e.b} = \text{const} > 0$ .

### Mathematical description of the problem solution

We write down the theorem on kinetic energy change for *non-free* material point on relocation  $AB$  between which the movement of the car is possible taking into account the initial velocity  $v_{i.br}$ , [19,20] in a finite form [12,13,14,6,7.] as applied to the solution of the problem under consideration by way of:

$$\frac{G}{2g}(v_{e.br}^2 - v_{i.br}^2) = A_{Fx}, \quad (1)$$

taking into account that in it:

$$A_{Fx} = A_{Gx} + A_{Ffr}, \quad (2)$$

where

$A_{Gx}$  – is the work of the gravity projection  $G_x$  along axis  $Ox$ , carried out parallel to the rail tracks on relocation  $x_{Cw}$  between points **A** and **B** executed by the force  $G_x$ :

$$A_{Gx} = G_x x_{Cw} = G \sin \psi_{bri} x_{Cw}; \quad (3)$$

$A_{Ffr}$  – работа силы трения  $F_{fr}$  (в общем случае, может быть и силы сопротивлений всякого рода  $F_r$ ) на

$A_{Ffr}$  – is the work friction force  $F_{fr}$  (in a general case they can be resistance forces  $F_r$ ) on relocation  $x_{Cw}$  between points **A** and **B**:

$$A_{Ffr} = -F_{fr} x_{Cw} = -f_{fr} G \cos \psi_{bri} x_{Cw}. \quad (4)$$

Substituting the last two formulas in (2) and taking into account (1), after simplification it is possible to obtain a formula of the car velocity in a braking zone on the sections of braking positions

$$v_{e.bri}^2 - v_{i.bri}^2 = 2g(\sin \psi_{bri} - f_{fr} \cos \psi_{bri}) x_{Cwi},$$

or at  $x_{Cwi} = l_{bi}$

$$v_{e.bri}^2 - v_{i.bri}^2 = 2g(\sin \psi_{bri} - f_{fr} \cos \psi_{bri}) l_{bri}, \quad (5)$$

Hence, at  $v_{e.bri} = 0$  и  $x_{Cwi} = l_{bri}$ ,

$$0 = v_{i.bri}^2 + 2g(\sin \psi_{bri} - f_{fr} \cos \psi_{bri}) l_{bri}.$$

From the last equation, we finally get a car braking path  $x_{Cwi} = l_{bri}$ :

$$l_{bri} = \frac{v_{i.bri}^2}{2g(f_{fr} \cos \psi_{bri} - \sin \psi_{bri})}. \quad (6)$$

Taking into consideration the fact that narrow angles (less than  $5^\circ$ ) corresponding the profile along the whole length of the yard hump path:  $\sin \psi_i \approx \psi_i = i$ ,  $\cos \psi_i$  then formulas (5) and (6) are as follows:

$$v_{e.bri}^2 = v_{i.bri}^2 + 2g(i - f_{fr}) l_{bri}, \quad (7)$$

$$l_{bri} = \frac{v_{i.bri}^2}{2g(f_{fr} - i)}. \quad (8)$$

As can be seen, the value of braking path  $l_{bri}$  is directly proportional to the square of the initial velocity  $v_{ibr}$  and inversely proportional to the coefficient of sliding friction  $f_{fr}$  and path profile grade  $i$ .

Thus, the application of the theorem on the change of the kinetic energy of a material point in a finite form {12,13} in the car braking zones on the sections of the braking positions according to formula (6) and/or (8) made it possible to determine car braking path  $l_{bri}$ .

In this case for calculating  $l_{bri}$ , the following options are considered:

a) direct entry to the section of the braking position of the leading wheel pair  $l_{bri}$  and/or wheel pairs of the leading truck  $l_{bt}$ ;

b) car entry to the section for the car basic length  $l_{ubr}$ , which are necessary for presetting initial velocity  $v_{i.br}$  and/or car entry velocity  $v_{e.br}$  (bearing in mind that  $v_{i.br} = v_{e.br}$ ) to the braking zone.

It is interesting to note that if the value of motion acceleration under uniformly decelerated movement according to the force correlation is known to be  $|a_{br}| = -a_{br}$  then using the elementary physics formula

$$v_{e.br}^2 = v_{i.br}^2 + 2|a_{br}|l_{br} , \quad (9)$$

it is possible to find braking time  $t_{br}$  before the complete stop of the car  $t_{br} < t$ , where  $t$  is current time in seconds

$$t_{br} = \frac{v_{i.br} - v_{e.br}}{|a_{i.br}|} . \quad (10)$$

For comparison by braking time value  $t_{br}$  it is possible to find car braking path  $l_{br}$  with the help of elementary physics formula on the survey section of braking positions.

Thus, the elementary physics velocity formulas under the given acceleration values in case of uniformly decelerated movement  $|a_{br}|$ , obtained by force correlations made it possible to find car braking time  $t_{br}$  before the car stops, i.e. under  $t_{br} < t$ , where  $t$  is current time in seconds.

It should be specified that car braking time  $t_{br}$ , calculated by the formula (10) has a negative sign. It points to slowing down of the car on the sections of braking positions and to the fact that  $t_{br} < t$  ( $t$  is current time)

**Initial data and results.** As an example, we consider the section of the hump second braking position (2ndBP). Initial data: for narrow angles  $\sin\psi_{2br} \approx \psi_{2br} = 0,010$  is track profile grade, rad., or  $i_{2br} = 10 \%$ ;  $G = 650$  is gravity force of the cargo on the car, kN;  $G_1 = 794$  is gravity force of the car with the cargo together with non-rotating parts (car body, truck, wheel pairs), kN;  $F_{x2} = 11,13$  with the regard for fair wind of small value ( $F_{w.x} = 3,2$  kN), κH;  $|F_{r2}| = -F_{r2} \approx -222,84$  is the module of resistance force of all kinds (taking into account the pressure force of retarder brake shoes to the wheel rim at the velocity of the car entry in the braking zone  $v_{entr.br2} \leq 6,5$  m/s;  $M_{ro} = 8,869 \cdot 10^4$  is reduced mass of the car with non-rotating parts, kg;  $v_{ibr2} = v_{entr.br2} = 3,879 \approx 3,88$  is initial velocity and/or car entry velocity in the braking zone of section (2ndBP), m/s.

**Calculation results** [37]. Car braking path on the hump section of the second braking position  $l_{br2}$ , calculated according to formula (6) proved to be  $l_{br2} = 3,195$  m. At that, there actually occurs a complete stop of the car, i.e.  $v_{br2} = 0$ .

Calculated acceleration value under car uniformly decelerated movement  $|a_{br}|$ , obtained by force correlations  $F_{x2}$  and  $|F_{r2}$  proved to be  $|a_{br}| = -2,387$  m/s<sup>2</sup>. In calculating car motion velocity  $v_{br2}$  according to elementary physics velocity formula (9) under  $l_{br2} = 3,195$  m and initial velocity  $v_{i.br2} = 3,879$  m/s there has been achieved a complex result:  $v_{b2} = I(v_{b2}) = 0,456i$  m/s, where  $i$  is an imaginary unit. It means that the actual part is  $R(v_{b2}) = 0$ . In its turn, that proves the correctness of derivation of analytical formula (6) and/or (8).

Car braking time according  $t_{br2}$  to elementary physics velocity formula (10) is  $t_{br2} = 1,625$  s before the moment of the car stop, when  $v_{br2} = 0$ , i.e. at  $t_{br2} < t$ , where  $t$  is current time in seconds. For example, at  $t = 1$  s  $v_{br2} = 1,492$  m/s; at  $t = 1,2$  s:  $v_{br2} = 1,05$  m/s; at  $t = 1,4$  s:  $v_{br2} = 0,535$  m/s; at  $t = 1,5$  s:  $v_{br2} = 0,299$  m/s; at  $t = 1,6$  s:  $v_{br2} = 0,06$  m/s.

As is evident, formula (9) can be used for determination of car sliding velocity before the moment of the complete car stop in the braking zone on the section of the hump braking positions.

The value of car braking path  $l_{br1}$ , defined according to elementary physics velocity formula on the survey section of the braking positions at braking time  $t_{br2} = 1,625$  s, proved to be  $l_{br02} = 3,152$  m. The relative calculation error as compared to formula (6) was 1,37 %, which is negligible.

Graphical dependence  $l_{br2} = f(v_{02br})$ , constructed according to formula (6) and to elementary physics formula (9) at variation  $v_{02br}$  from 0 to 5 with step  $\Delta v_{02br} = 0,25$  m/s, is presented in Fig.1.

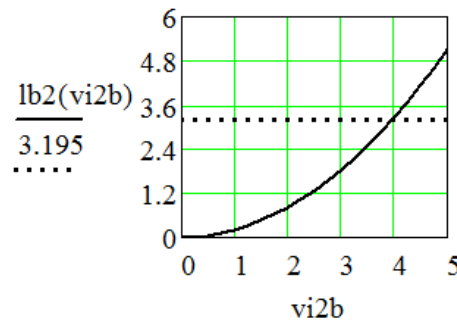


Fig. 1. Graphical dependence  $l_{br2} = f(v_{02br})$

As evident from Fig.1, the character of dependence of braking path upon the car initial velocity has the form of increasing squared relationship.

As may be seen, at  $v_{ibr2} = v_{entr.br2} = 0$  braking path  $l_{br2} = 0$ . This validates the statement about the importance of car entry onto the braking zone of braking positions at initial velocity  $v_{ibr2} = v_{entr.br2} > 0$ , otherwise there occurs complete stop of the car before car retarder activation. Under meeting the condition  $v_{ibr2} = v_{entr.br2} > 0$  kinetic energy  $E_k = E_0$  of the car with mass  $M$  and initial velocity  $v_{ibr2}$  will be completely executed on work  $A_r$  for overcoming resistance forces  $F_r$ , ensuing from the car retarder activation. Under complete car stop, i.e.  $v_{e.br2} = 0$ , the condition:  $E_0 + (-A_r) = 0$  will be fulfilled.

Thus, the results of calculation of car braking path by using formula (6) elementary physics path formula made it possible to conclude that at one and the same value of initial velocity  $l_{br2}$  they give similar results.

In its turn, this verifies indisputability, correctness and applicability of constructed mathematical models with reference to the car braking zone on all braking positions.

### Conclusion

1. On the basis of the theorem on the change of kinetic energy for a non-free material point there for first time has been solved the problem of mathematical modeling of braking zone of railway hump braking sections.

2. Calculation results of the car braking path based on the application of formulas (6) or (8) and elementary physics path formula put forward by the authors of the article made it possible to conclude that at the one and same value of the initial velocity they produce similar results.

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